SENRA Academic Publishers, British Columbia
Vol. 8, No. 3, pp. 3155-3161, October 2014
Online ISSN: 1920-3853; Print ISSN: 1715-9997

Canadian Journal of pure&applied sciences

SOIL MODULUS AND UNDRAINED COHESION OF CLAYEY SOILS FROM STRESS-STRAIN MODELS

*SB Akpila and IW Omunguye
Department of Civil Engineering, Rivers State University of Science and Technology
PM B 5080, Port Harcourt, Nigeria

ABSTRACT

A study based on the stress-strain behaviour of soils in four areas within Port Harcourt has been carried out. In this study, deformation trends on stress and strain, derivative of stress and strain, and ratio of deviator stress to undrained cohesion to strain were established. Higher stability and lower deformation of soils response to loading were in descending order of Rukpoku, Ada George, Borikiri and Abuloma areas. At low strains, soil modulus E, generally reduced with increase in strain converging towards 3.5% strain and subsequently, exhibited slight increase in value on Rukpoku and Ada George Road soils. Predicted soil moduli for the areas are generally within the range of E, identified as soft to medium clay soils, except for Rukpoku soils that are within the range of hard clay. Predicted values of the ratio of deviator stress to undrained cohesion and strain, at strain level of 1% are generally lower than reported field values frequently used for intact blue London clay, but are within the value used for routine work in London clay. Shallow foundation settlement input parameter of soil modulus can easily be obtained from the predictive models or values, for preliminary analysis and design.

Keywords: Deviator stress, undrained cohesion, deformation, foundation settlement.

INTRODUCTION

Soils are generally subjected to various loads, which in this context can be those from load bearing walls, columns, vehicular wheel loads, and machine foundations, etc, causing stresses in the soil mass. The soils correspondingly experience varying levels of strain. This relationship can be exemplified in the laboratory for soils subjected to, for instance, triaxial compression, or direct shear on fully saturated soils. Hence, it becomes imperative to understand the shape of stress-strain curves of these soils in general. Problems involving the application of stresses to soils may be divided into those in which (a) deformation of the foundation soil control design and (b) failure of the foundation soil controls design. In deformation-controlled design, the deformation of a mass of soil must be computed and this ensures that the shape of the stress-strain curve must be taken into account. But for failure-controlled design, the precise shape of the stress-strain curve need not be known if shear stress reaches a maximum and subsequently remains constant even at very large strain (Poulos, 1971). Literatures in stability analysis reports stress-strain curves of soils reaching a maximum shear stress which then undergoes constant deformation under continuous strain. But in the laboratory, compressive triaxial test are generally not continued to attain continuous deformation under shear stress. Four major factors are considered to significantly control the shape of stress-strain curves of soils; soil type, initial structure, initial state and method of loading. The sections of stress-strain curve are affected to varying degree by these factors; before the steady state, the stress-strain curve is affected by all the factors whereas in the steady state, the shear stress is not influenced by initial structure, the stress path followed during loading or the initial state. However, the strain required to attain the steady state may be dependent on method of loading, initial state, initial structure and soil type (Poulos, 1971). Soils exhibit nonlinear stress-strain curve, and different soil moduli can be deduced from the slope of the curve. Depending on where the slope is determined, the secant modulus, tangent modulus, unloading modulus or reload modulus can be obtained. Consequently, appropriate selection of these moduli in engineering applications is important. For instance, the secant modulus is appropriate in predicting spread footing movement due to first load application; the tangent modulus is used in cases of evaluating incremental movement due to incremental load from one more storey in a high–rise building. Also, the unloading modulus is useful in calculating heave at bottom of excavation or rebound on pavement on removal of truck tyre load, while reload modulus is used in calculating bottom excavation movement on replacement of excavated soil or equivalent overburden (Briaud, 2010). The classical elasticity modelassumes soil behaviour under loading to be elastic and under axi-symmetric loading, the following equations are expressed (Bolton, 1979):

*Corresponding author email: sakpilab@gmail.com

$$\epsilon_1 = \frac{1}{E}(\sigma_1 - \mu\sigma_2 - \mu\sigma_3) \quad (1)$$

$$\epsilon_2 = \frac{1}{E}(-\mu\sigma_1 + \sigma_1 - \mu\sigma_3) \quad (2)$$

$$\epsilon_3 = \frac{1}{E}(-\mu\sigma_1 - \mu\sigma_2 + \sigma_3) \quad (3)$$

Where σ_1, σ_2 and σ_3 are three dimensional stresses, $\epsilon_1, \epsilon_2,$ and ϵ_3 are three-dimensional strains, μ is poison ratio and E is modulus of soil.

Under uniaxial loading

($\sigma_2 = \sigma_3 = 0, and \ \epsilon_2 = \epsilon_3 = 0$) Equation (1) becomes;

$$E = \frac{\sigma_1}{\epsilon_1} \quad (4)$$

If the stress-strain curve is represented by $f(x)$, then incorporating Equation (4), the slope of the stress-strain curve can be expressed as;

$$\frac{d}{dx} f(x) = \frac{\sigma_1}{\epsilon_1} = E \quad (5)$$

Given that the deviator stress is $\sigma_1 - \sigma_3$ then the slope of

$(\sigma_1 - \sigma_3)/c_u$ versus axial strain, ϵ can be represented as

follows;

$$\frac{d}{d\epsilon}\{(\sigma_1 - \sigma_3)/c_u\} = \alpha \quad (6)$$

Where $\alpha = \frac{E}{c_u}$ and c_u = undrained cohesion.

Immediate settlement of shallow foundations placed on cohesive soils can be evaluated with value of undrained modulus, E_u, of the supporting soil as input parameter. However, determination of E_u is faced with several challenges and in Barnes (2000) and Jamiolkowski *et al.* (1979) proposed ratio of undrained modulus to undrained cohesion (E_u/c_u) depending on overconsolidation ratio and plasticity index. Butler (1974) proposed E_u/c_u ratio of 400 that is frequently used for intact blue overconsolidated London clay, while E_u/c_u ratio of 140 is proposed by Padfield and Sharrock (1983) for routine work in London clay. A procedure for obtaining undrained modulus directly from triaxial test results by determining the strain corresponding to 65% of the maximum deviator stress and dividing this value into its corresponding stress is outlined by Skempton (1951) and Smith (1982). In many literatures, E_u for various soils is presented in a wide range of values with little emphasis as to whether they are secant modulus, tangent modulus, unload modulus or reload modulus (Bowles, 1997). However, the initial tangent modulus is quite often used to represent the stress-strain modulus of a soil. This application is due to the elastic response of soils generally observed only near the origin and which is almost the same for different test plots. The adoption of re-load modulus has been emphasised as a better choice and it is generally higher than the initial tangent modulus of the first cycle due to the effect of strain hardening (Raj, 2008). Recent studies on the development of predictive models on evaluation of settlement parameters of void ratios e, coefficient of volumetric compressibility m_v, and compression modulus E_c, on clayey soils have been reported (Akpila, 2013a,b). It was observed that values of e, and m_v, generally showed a decreasing trend with increase in pressure, while E_c increased with pressure.

Based on the difficulty in evaluating the relevant soil modulus needed in the analysis and design of foundation, an attempt is made in this paper to develop a predictive model for the studied areas.

MATERIALS AND METHODS

Acquisition and Analysis of Data

A total of 81 unconsolidated undrained triaxial test results were analysed from each of the four areas studied in Port Harcourt: Abuloma, Ada George Road, Rukpoku and Borikiri**.** The deviator stresses, induced strains, cross-sectional area, major$(\sigma_1)$and minor (σ_3)principal stresses were evaluated. For instance, the deviator stress ($\sigma_{1-}\ \sigma_3$), is evaluated noting that the average cross-sectional area (A) of the specimen does not remain constant throughout the test. When the original cross-sectional area of the specimen is A_o and the original volume is V_o then, for a decrease in volume of the specimen during the test, the average cross-sectional area (A) is expressed as;

$$A = A_o \frac{1 - \varepsilon_v}{1 - \varepsilon_a} \quad (7)$$

If the volume of specimen increases during the test, then Equation (1) becomes;

$$A = A_o \frac{1 + \varepsilon_v}{1 - \varepsilon_a} \quad (8)$$

Where ε_v is the volumetric strain ($\Delta v/v_o$), and ε_a is the axial strain ($\Delta l/l_o$). Deviator stresses were evaluated by dividing the load with corresponding cross-sectional area of the sample.

RESULTS AND DISCUSSION

Deviatorstress - strain curve

The various nonlinear stress-strain curves and soil modulus-strain curves of clayey soils from four studied areas are shown in figures 1- 4. The variation of applied deviator stress to strain induced on the soils is observed to have increased in the order of soils obtained from Abuloma, Borokiri, Ada George Road and Rukpoku. This is indicative of the tendency of soils within Rukpoku to exhibit higher stability and lower deformation as against the response of loading to stability and deformation on soils within the three other areas. Soils within Ada George Road showed middle bound response to loading, while soils within Borokiri have higher stability compared to those within Abuloma. Their predictive trends are given by Equations (9-12). For cell pressure of 300 kN/m^2, the stress-strain curves are represented by Equations (13-16). The failure curve is typical of the non-linear behaviour of soils under deformation and the stress magnitudes sustained by the soils are highest on Rukpoku soils and lowest on Abuloma soils. Soils within Ada George Road had middle bound values, while those of Borikiri were higher than those from Abuloma.

Soil Modulus

The slope of stress-strain curves of Equations (9-12) are presented in Equations (17-20) while the variation of soil modulus E, with strain for cell pressure of 100 kN/m^2 is presented in figure 3. Soil modulus E, generally had a decreasing trend, with maximum values obtained at zero strain; Rukpoku soils had highest values, while Abuloma soil had lowest E. For strains exceeding 3%, soil modulus of Rukpoku soils showed a characteristic increase in value.

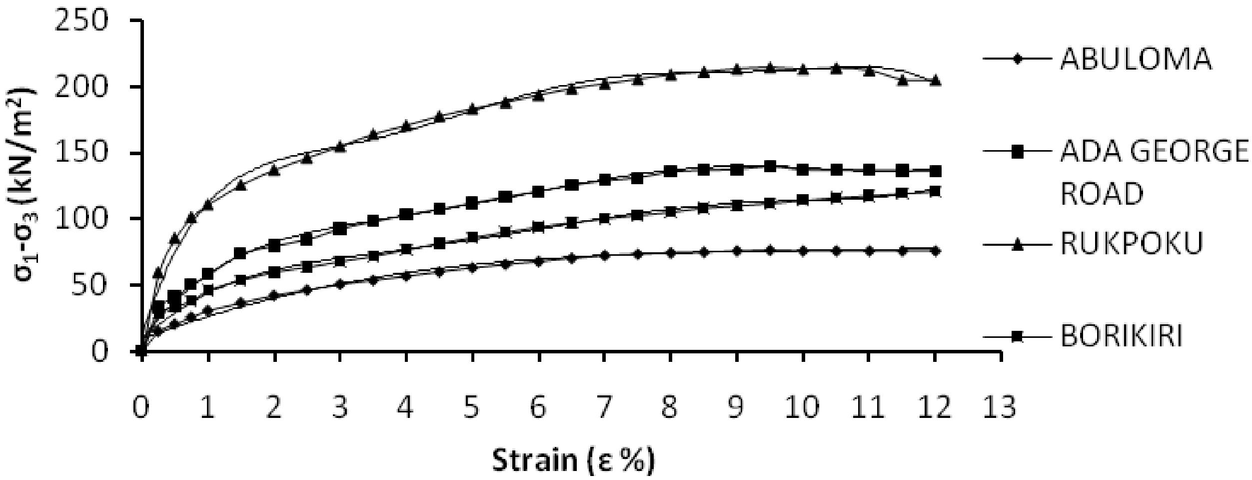

Fig. 1. Deviator stress and strain (σ_3 =100kN/m^2).

$\sigma_1 - \sigma_3$ = - 0.004ε^6 + 0.169ε^5 - 2.634ε^4 + 20.09ε^3 - 78.35ε^2 + 159.8ε + 13.44; R^2 = 0.991	Rukpoku	(9)
$\sigma_1 - \sigma_3$ = 0.065ε^3 - 1.909ε^2 + 19.08ε + 9.137; R^2 = 0.986	Abuloma	(10)
$\sigma_1 - \sigma_3$ = 0.007ε^5 - 0.234ε^4 + 2.846ε^3 - 15.88ε^2 + 47.99ε + 9.233; R^2 = 0.992	Borokiri	(11)
$\sigma_1 - \sigma_3$= 0.009ε^5 - 0.325ε^4 + 3.9816ε^3 - 22.54ε^2 + 68.11ε + 9.005; R^2= 0.994	Ada George	(12)

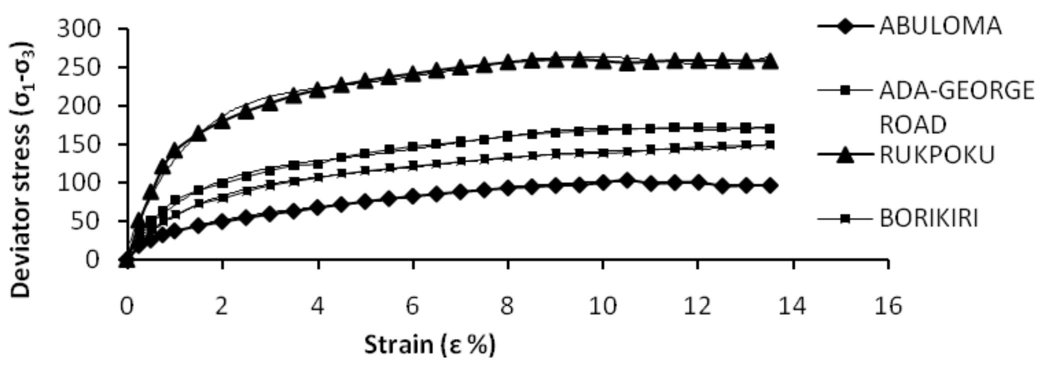

Fig. 2. Deviator Stress and Strain (σ_3 = 300 kN/m^2).

$\sigma_1 - \sigma_3$ = 0.013ε^5 - 0.496ε^4 + 7.037ε^3 - 46.85ε^2 + 155.1ε + 15.44; R^2 = 0.992	Rukpoku	(13)
$\sigma_1 - \sigma_3$ = 0.006ε^5 - 0.237ε^4 + 3.374ε^3 - 22.48ε^2 + 78.76ε + 12.34; R^2 = 0.993	Ada George	(14)
$\sigma_1 - \sigma_3$ = 0.003ε^5 - 0.149ε^4 + 2.207ε^3 - 15.70ε^2 + 60.56ε + 9.673; R^2 = 0.995	Borokiri	(15)
$\sigma_1 - \sigma_3$ = 0.003ε^5 - 0.115ε^4 + 1.597ε^3 - 10.26ε^2 + 37.58ε + 6.408; R^2 = 0.994	Abuloma	(16)

$$E = -0.024\varepsilon^5 + 0.845\varepsilon^4 - 10.536\varepsilon^3 + 60.27\varepsilon^2 - 156.7\varepsilon + 159.8;\ R^2 = 0.991 \quad \text{Rukpoku} \quad (17)$$
$$E = 0.195\varepsilon^2 - 3.818\varepsilon + 19.08;\ R^2 = 0.986 \quad \text{Abuloma} \quad (18)$$
$$E = 0.035\varepsilon^4 - 0.936\varepsilon^3 + 8.538\varepsilon^2 - 31.76\varepsilon + 47.99;\ R^2 = 0.992 \quad \text{Borokiri} \quad (19)$$
$$E = 0.045\varepsilon^4 - 1.3\varepsilon^3 + 11.944\varepsilon^2 - 45.08\varepsilon + 68.11;\ R^2 = 0.992 \quad \text{Ada George} \quad (20)$$

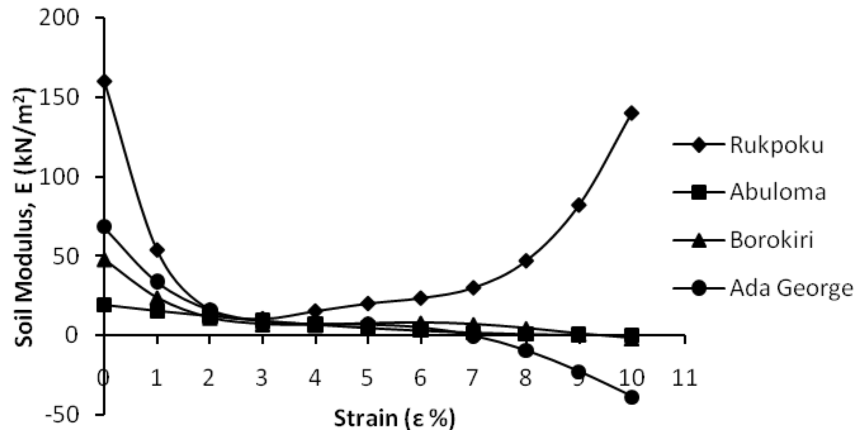

Fig. 3. Soil Modulus and Strain ($\sigma_3 = 100$ kN/m^2).

$$E = 0.065\varepsilon^4 - 1.96\varepsilon^3 + 21.11\varepsilon^2 - 93.7\varepsilon^2 + 155.1;\ R^2 = 0.992 \quad \text{Rukpoku} \quad (21)$$
$$E = 0.030\varepsilon^4 - 0.948\varepsilon^3 + 10.12\varepsilon^2 - 44.96\varepsilon + 78.76;\ R^2 = 0.993 \quad \text{Ada George} \quad (22)$$
$$E = 0.015\varepsilon^4 - 0.596\varepsilon^3 + 6.621\varepsilon^2 - 31.4\varepsilon + 60.56;\ R^2 = 0.995 \quad \text{Borokiri} \quad (23)$$
$$E = 0.015\varepsilon^4 - 0.46\varepsilon^3 + 4.791\varepsilon^2 - 20.52\varepsilon + 37.5;\ R^2 = 0.994 \quad \text{Abuloma} \quad (24)$$

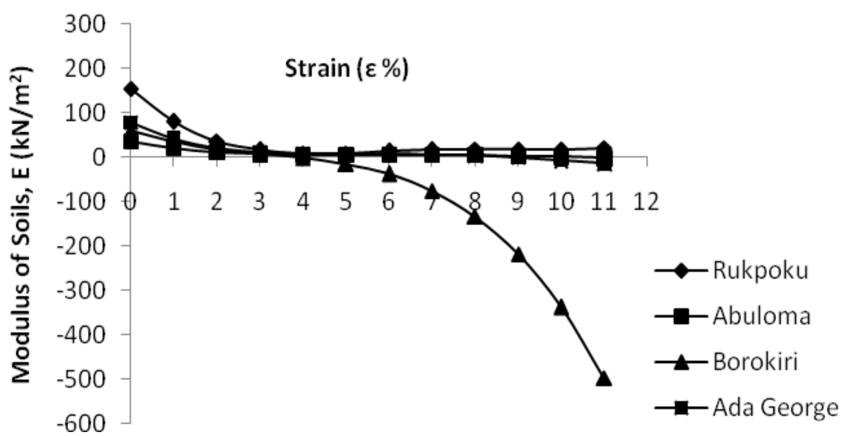

Fig. 4. Variation of Modulus with Strain ($\sigma_3 = 300$ kN/m^2).

The rate of change of stress with strain, expressed by the soil modulus, for cell pressure of 300kN/m^2 is presented in Equations (21-24) and depicted in figure 4. At low strains, soil modulus generally reduced with increase in strain converging towards 3.5% strain. Thereafter, E exhibited slight increase in value on Rukpoku and Ada George Road soils.

Stress to Undrained cohesion, (σ_1 - σ_3)/c_u, and Strain

Soils response in terms of the ratio of deviator stress to undrained cohesion, (σ_1 - σ_3)/$c_u$and strain for cell pressure of 100 and 300 kN/m^2 are presented in figures 5 and 6, respectively. The response trend is generally non-linear with highest values found on soils within Borikiri, and lowest values on Abuloma soils. Middle bound value is

$(\sigma_1 - \sigma_3)/c_u = -0.011\varepsilon^4 + 0.135\varepsilon^3 - 0.755\varepsilon^2 + 2.284\varepsilon + 0.435;\ R^2 = 0.992$ Borokiri (25)

$(\sigma_1 - \sigma_3)/c_u = -0.009\varepsilon^4 + 0.115\varepsilon^3 - 0.660\varepsilon^2 + 1.905\varepsilon + 0.338;\ R^2 = 0.984$ Rukpoku (26)

$(\sigma_1 - \sigma_3)/c_u = -0.005\varepsilon^4 + 0.068\varepsilon^3 - 0.391\varepsilon^2 + 1.189\varepsilon + 0.155;\ R^2 = 0.994$ Ada George Rd (27)

$(\sigma_1 - \sigma_3)/c_u = -0.004\varepsilon^4 + 0.052\varepsilon^3 - 0.305\varepsilon^2 + 1.010\varepsilon + 0.117;\ R^2 = 0.996$ Abuloma (28)

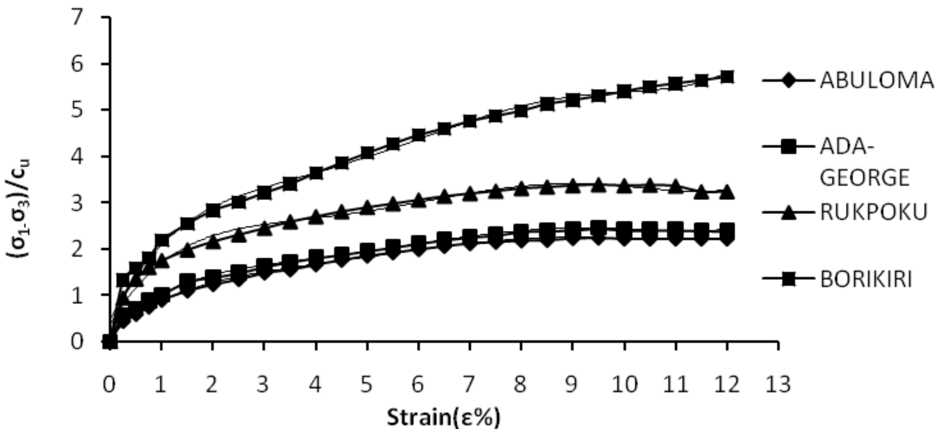

Fig. 5. Deviator stress to undrained cohesion and strain (σ_3 =100 kN/m^2).

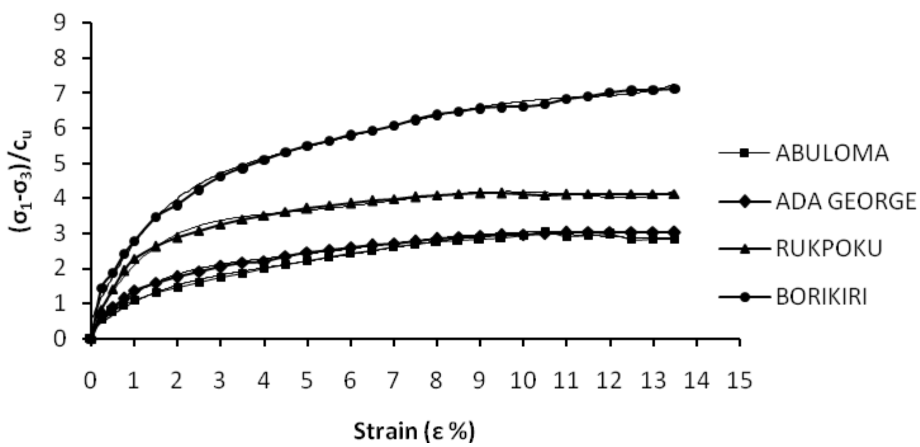

Fig. 6. Deviator Stress to Undrained cohesion and Strain (σ_3 = 300kN/m^2).

$(\sigma_1 - \sigma_3)/c_u = -0.007x^4 + 0.105x^3 - 0.747x^2 + 2.884x + 0.460;\ R^2 = 0.995$ Borokiri (29)

$(\sigma_1 - \sigma_3)/c_u = -0.007x^4 + 0.111x^3 - 0.743x^2 + 2.463x + 0.245;\ R^2 = 0.992$ Rukpoku (30)

$(\sigma_1 - \sigma_3)/c_u = -0.004x^4 + 0.059x^3 - 0.394x^2 + 1.381x + 0.216;\ R^2 = 0.993$ Ada George Rd (31)

$(\sigma_1 - \sigma_3)/c_u = -0.003x^4 + 0.047x^3 - 0.301x^2 + 1.105x + 0.188;\ z^2 = 0.994$ Abuloma (32)

associated with Rukpoku soils but soils around Ada George Road had values that are slightly higher than those within Abuloma. Their respective trend lines are given by Equations (25-28).

MODEL VERIFICATION

Soil Modulus

Evaluation of tangent modulus of the soils at 1% strain level for cell pressures of 100 kN/m^2 and 300 kN/m^2 based on deviator stress-strain models of Equations (9-12) and (13-16) respectively are shown in table1. The predicted soil modulus are generally in the range of E, identified as soft to medium clay soils, except for Rukpoku soils that are in the range of hard clay. From the derivatives of the deviator stress-strain models of Equations (17-20) and (21-24), the soil modulus at 1% strain level are also presented in table 2. The predicted soil moduli for the areas suggested Abuloma soils as soft

Table 1. Model verification of soil modulus (Stress-strain models).

Location	Predicted Tangent Modulus E (MPa)	E from field values (MPa)	
		Clay soil	Range
Rukpoku	99-115	Very soft	2-15
Ada George Rd	45-59	Soft	15-25
Borikiri	35-47	Medium	25-50
Abuloma	17-29	Hard	50-100

Table 2. Model verification of soil modulus ($\frac{d}{d\varepsilon}(\sigma_1 - \sigma_3)$).

Location	Predicted Tangent Modulus E (MPa)	E from field values (MPa)	
		Clay soil	Range
Rukpoku	54-81	Very soft	2-15
Ada George Rd	23-43	Soft	15-25
Borikiri	16-43	Medium	25-50
Abuloma	15-21	Hard	50-100

clays, those within Borikiri and Ada George Road as soft to medium clays, but Rukpoku having E of hard clays.

Stress to Undrained cohesion, $(\sigma_1 - \sigma_3)/c_u$and Strain

The predicted values of E/c_u based on the ratio of deviator stress to undrained cohesion, $(\sigma_1 - \sigma_3)/c_u$, and strain for cell pressure of 100 and 300 kN/m^2 at strain level of 1% is presented in Tables 3. Predicted E/c_u values are generally lower than reported field values frequently used for intact blue London clay, but are within the value used for routine work in London clay.

CONCLUSION

The stress-strain curve of the soils showed nonlinear deformation behaviour and the predicted soil modulus for the areas are generally in the range of E identified as soft to medium clay soils, except for Rukpoku soils that are within the range of hard clays.

Based on the derivatives of the deviator stress-strain models, Abuloma soils have E described as soft clays, those within Borikiri and Ada George Road have E associated with soft to medium clays, but Rukpoku area has E described as hard clays. The predicted values of E/c_u based on the ratio of deviator stress to undrained cohesion, $(\sigma_1 - \sigma_3)/c_u$and strain at strain level of 1% generally gave values lower than reported field values frequently used for intact blue London clay, but are within the values used for routine work in London clay.

Ultimately, foundation settlement input parameter of soil modulus in the studied areas can easily be obtained from generated predictive models or values, for purposes of preliminary analysis and design of shallow foundations placed on cohesive soils.

REFERENCES

Akpila, SB. 2013[a]. Predictive Approach on Evaluation of Settlement Parameters on Clayey Soils in Parts of Port Harcourt. Scientific Journal of Pure and Applied Sciences. 2(2):66-71.

Akpila, SB. 2013[b]. Predictive Models on Settlement Parameters of Clayey Soils: A Case Study in Port Harcourt City of Nigeria. Canadian Journal of Pure and Applied Sciences. 7(3):2649-2653.

Barnes, GE. 2000. Soil Mechanics, Principles and Practice. (2nd edi.). MacMillan Press Ltd, London. pp267.

Bolton, M. 1979. Guide to Soil Mechanics. The MacMillan Press Ltd, London. pp120.

Bowles, JE. 1997. Foundation Analysis and Design. (5th edi.). MacGraw- Hill International Editions. pp125.

Briaud, JL. 2010. Introduction to Soil Moduli. Static2.docstoccdn.com/docs/42184625/1

Butler, FG. 1974. Review Paper: Heavily Over-consolidated Clays. In: Proceedings of the Conference on Settlement of Structures. Pentech Press, Cambridge. 531-578.

Jamiolkowski, M., Lancellotta, R., Pasqualini, E., Marchetti, S. and Nava, R. 1979. Design Parameters for Soft Clays. General Report. Proc. 7th European Conf. on Soil Mechanics and Foundation Engineering. 5:27-57.

Padfield, CJ. and Sharrock, MJ. 1983. Settlement of Structures on Clay Soils. Construction Industry Research and Information Association. Special Pub. pp27.

Poulos, CJ. 1971. Stress-Strain Curves of Soils, Geotechnical Engineers, Inc. Winchester, Massachusetts. 01890.

Raj, PP. 2008. Soil Mechanics and Foundation Engineering. Dorling Kindersley Pvt Ltd., India. pp171.

Skempton, AW. 1951. The Bearing Capacity of Clays. Building Research Congress.

Smith, GN. 1982. Elements of Soil Mechanics for Civil and Mining Engineers. (5th edi.). Billing and Sons Ltd, UK. 348-349.

Received: June 27, 2014; Revised and Accepted: Aug 1, 2014